



Računarska grafika

predavanja
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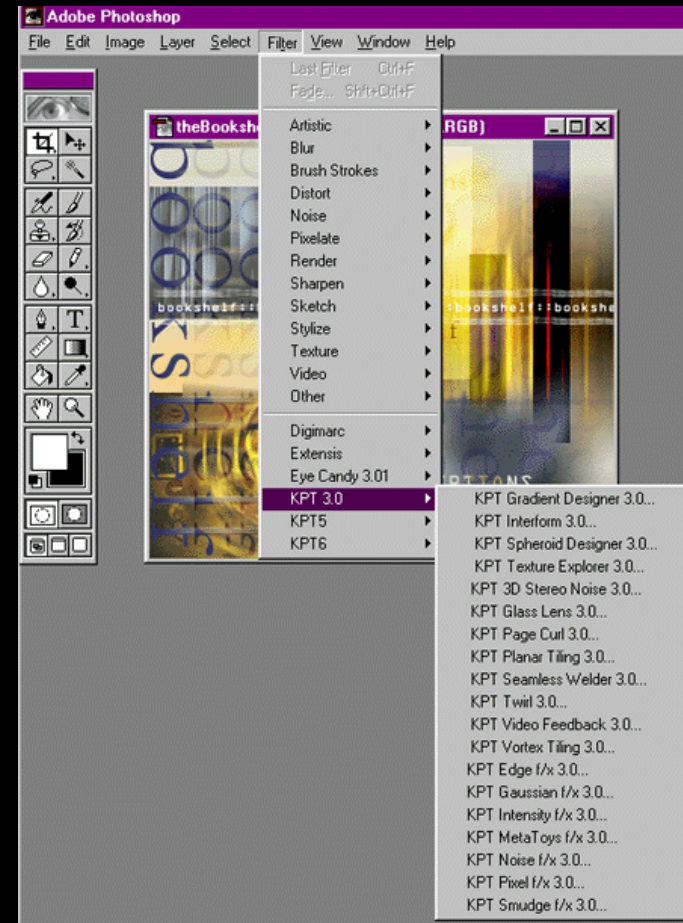
Filteri

- Konvolucijski/linearni filteri
- Gauss filteri
- Omekšavanje (smoothing) i smanjenje šuma (noise reduction)
- Orijentisani Gauss filteri
- Derivacija Gauss filtera
- Detekcija ivica



Filteri

- Za obradu digitalne slike koriste se filteri
- Mogu biti ugrađeni u software ili naknadno instalirani, kao add-ins
- Filtriranje koristi matematičke funkcije za transformaciju rasterske slike (proračun boje pojedinačnih piksela)



Filteri

- Koriste se za korekciju grešaka u slici, imitiranje slikarskih tehnika, postizanje različitih efekata, vektorizaciju,...



Konvolucijske formule

- 1D formula:

$$h(x) \cdot f(x) = \int_u h(x-u) f(u) du$$

$$h(x) \cdot f(x) = \sum_i h(x-i) f(i)$$

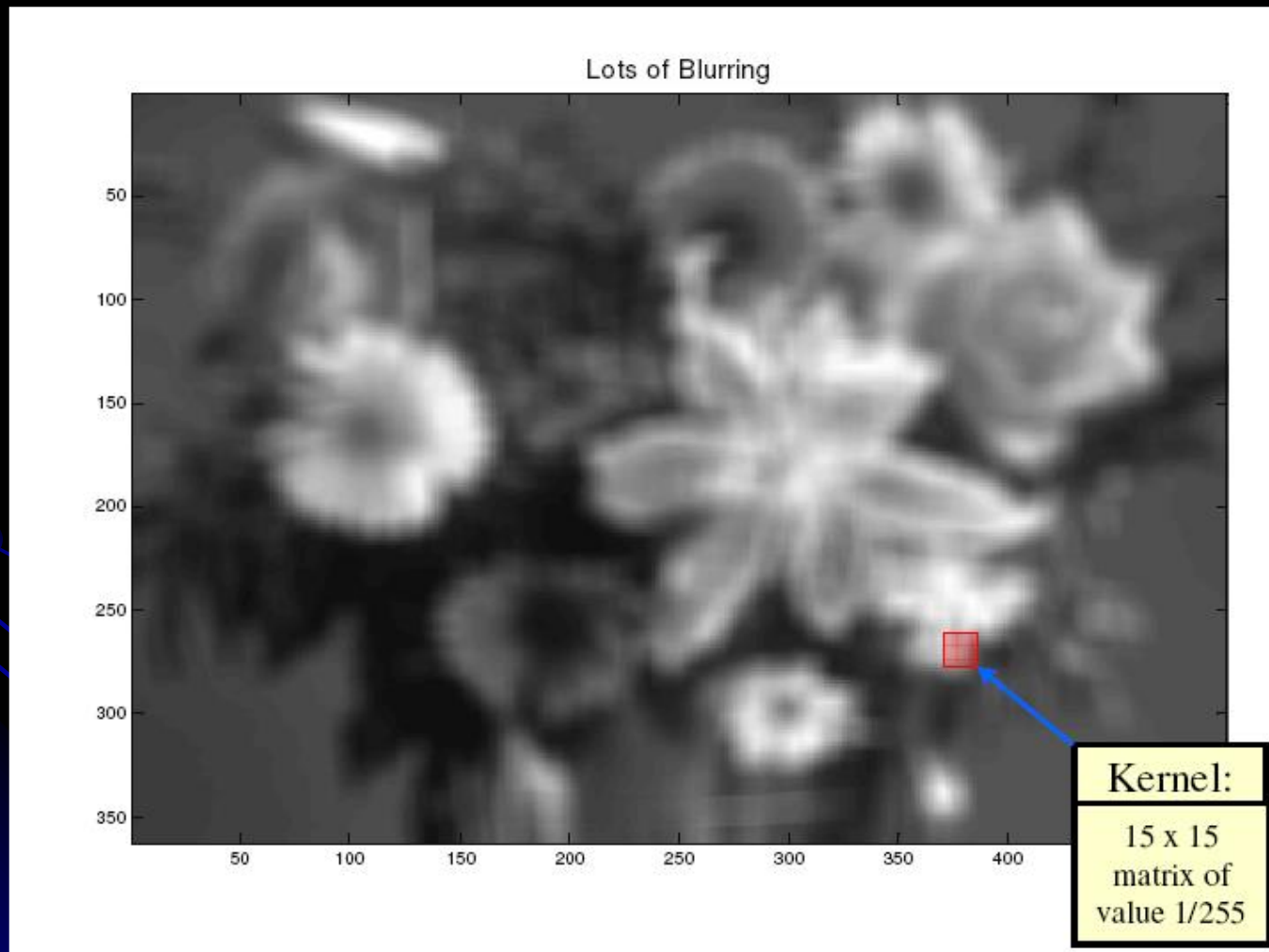
- 2D formula:

$$h(x, y) \cdot f(x, y) = \int_u \int_v h(x-u, y-v) f(u, v) du dv$$

$$h(x, y) \cdot f(x, y) = \sum_i \sum_j h(x-i, y-j) f(i, j)$$

- Konvolucija mijenja boju piksela na osnovu boje piksela koji ga okružuju
- Primjer: <http://www.roborealm.com/help/Convolution.php>

Konvolucijski filter



Osnovne osobine

- Komutativnost: $f * g = g * f$
- Asocijativnost: $(f * g) * h = f * (g * h)$
- Linearnost: $(a * f + b * g) * h = a * f * h + b * g * h$
- Invarijantnost: $f_t * h = (f * h)_t$
- Diferencijabilnost:

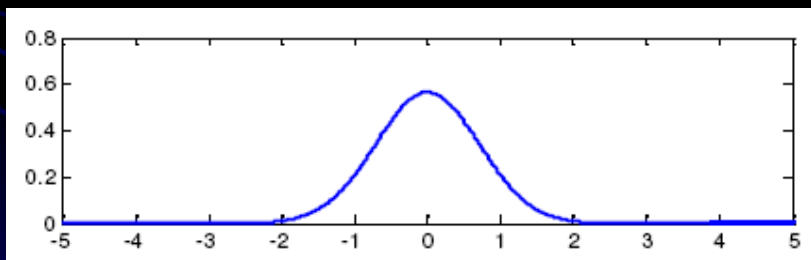
$$\frac{\partial}{\partial x}(f * g) = \frac{\partial f}{\partial x} * g$$

Gauss filteri

- Gauss operator za omekšavanje slike je 2D konvolucijski operator koji se koristi da "zamuti" sliku i ukloni detalje i šum.

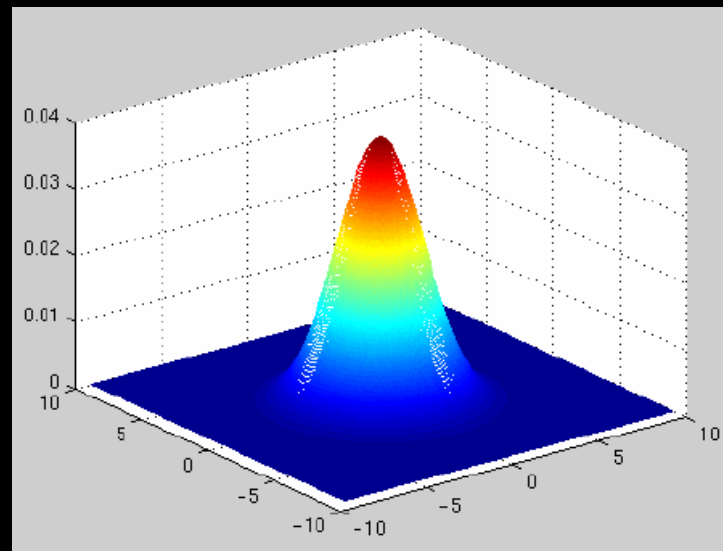
- 1D:

$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$

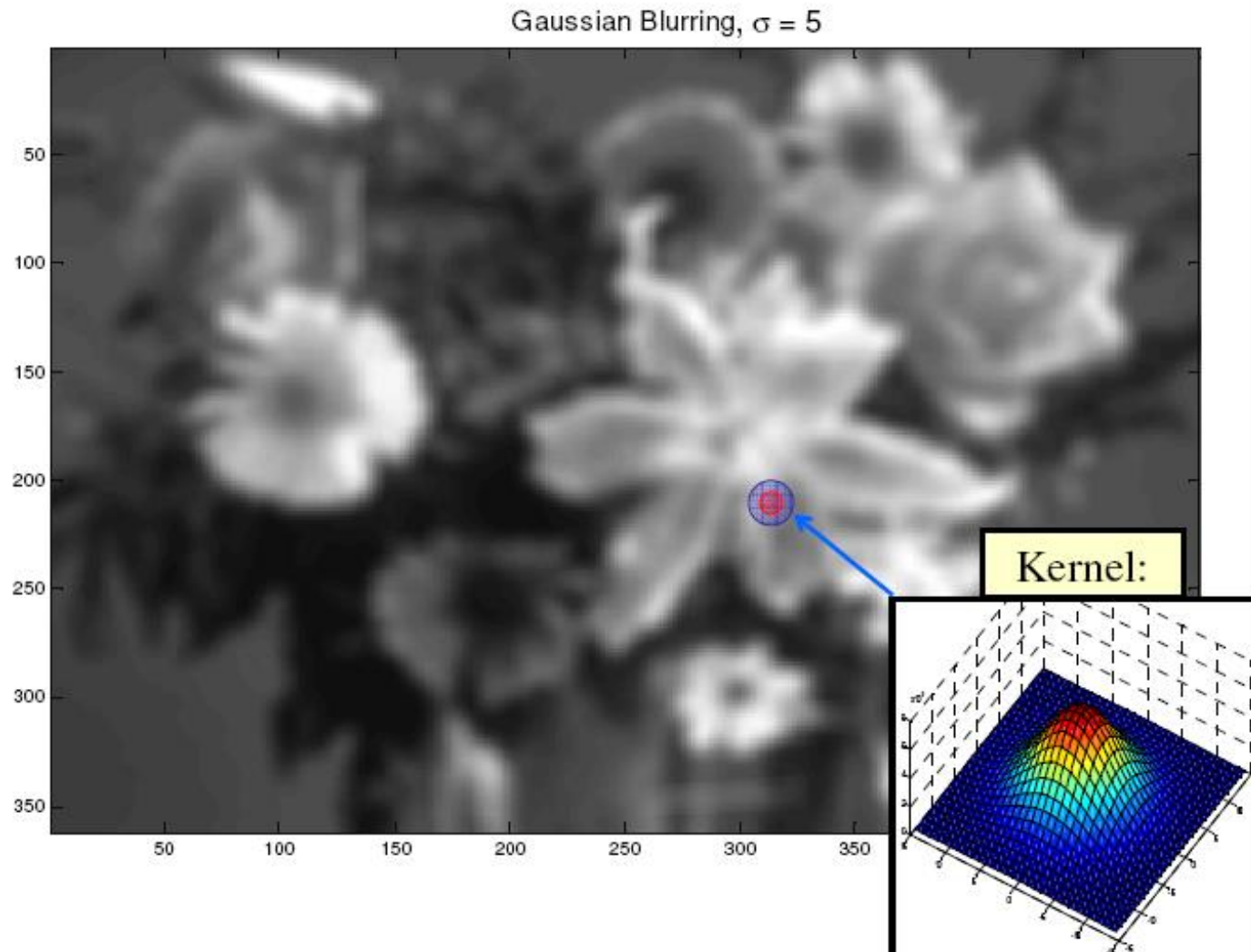


- 2D:

$$G(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

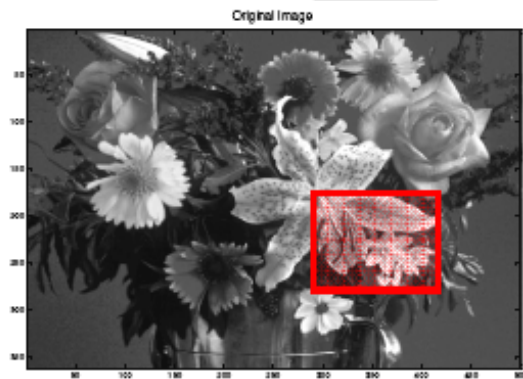
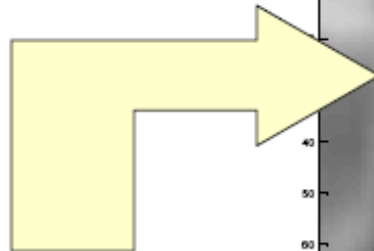


Gauss filteri

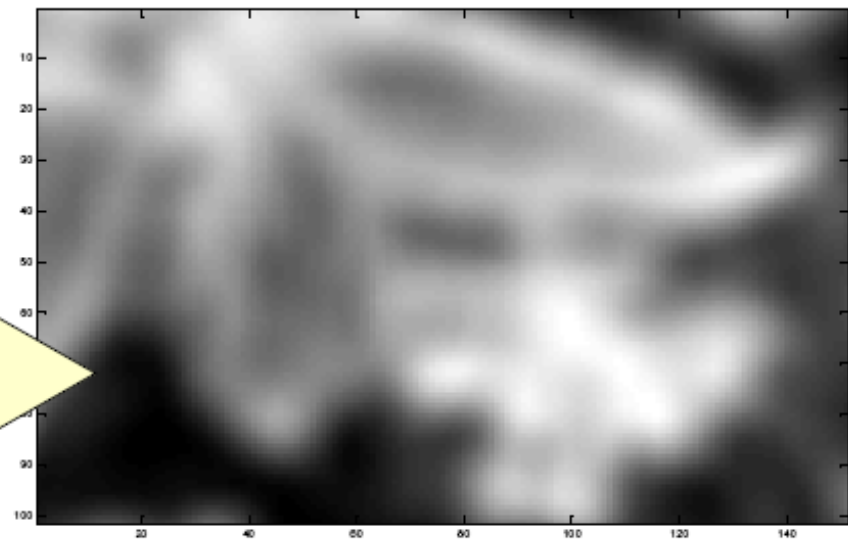
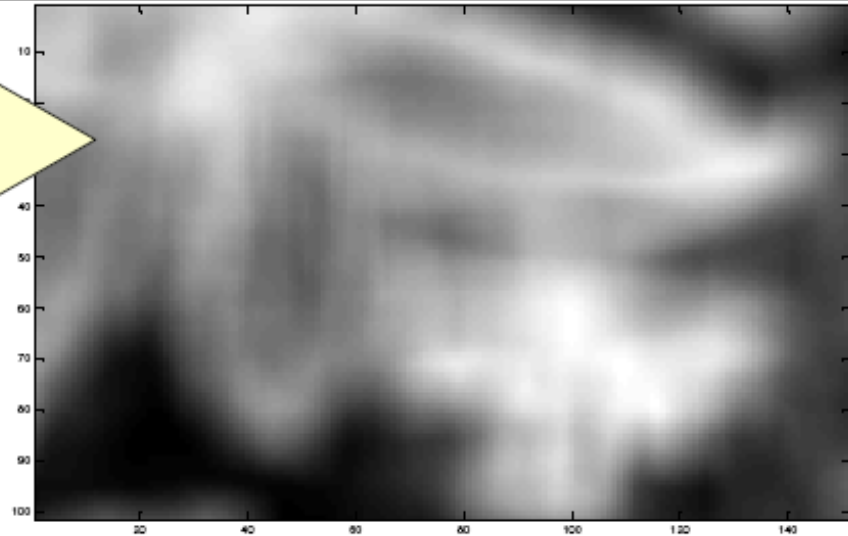
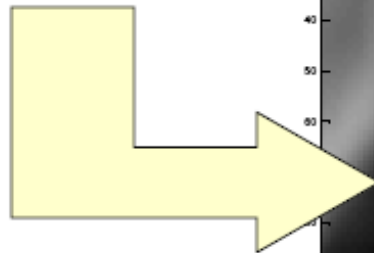


Gauss filteri

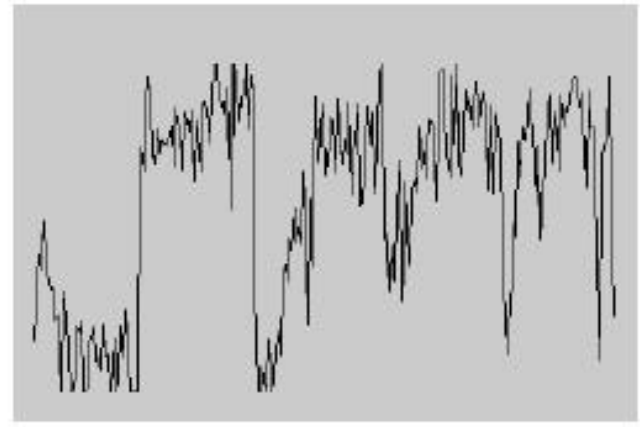
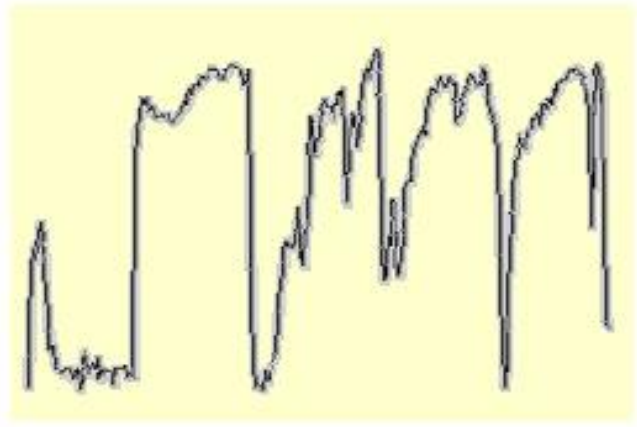
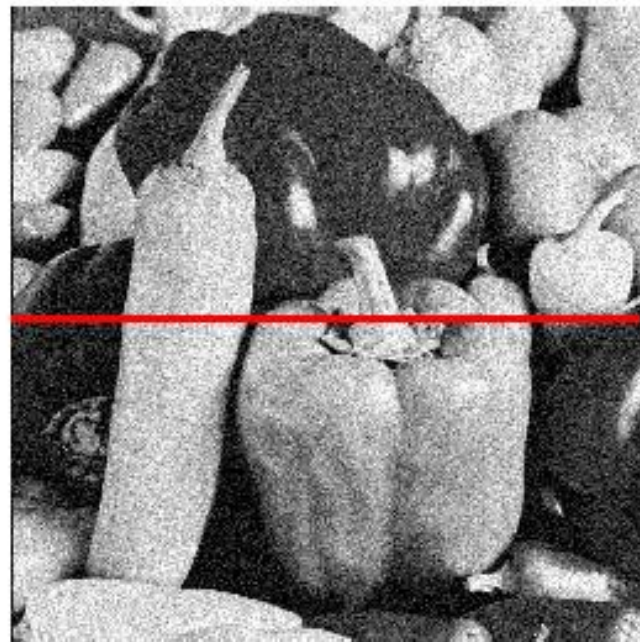
Obična
prosječna
vrijednost



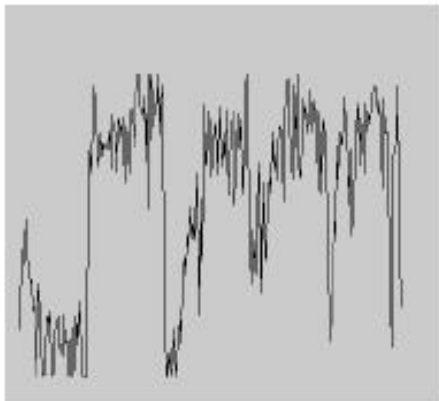
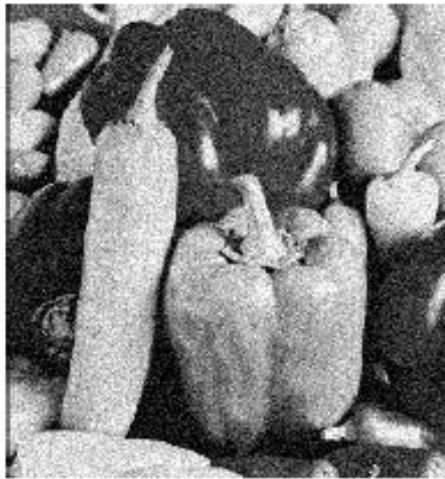
Gauss
"mekšanje"



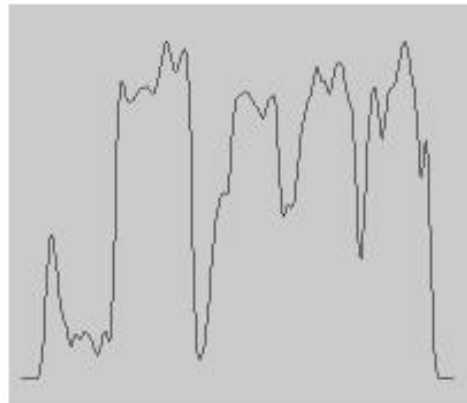
Šum slike



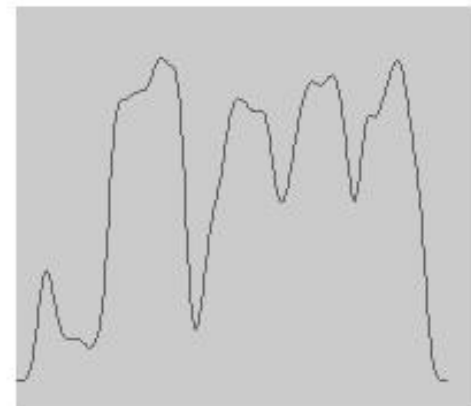
Uklanjanje šuma Gauss filterom



Bez filtera

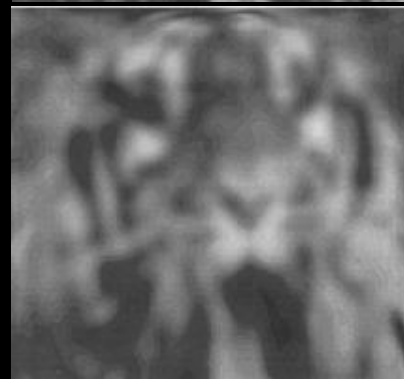
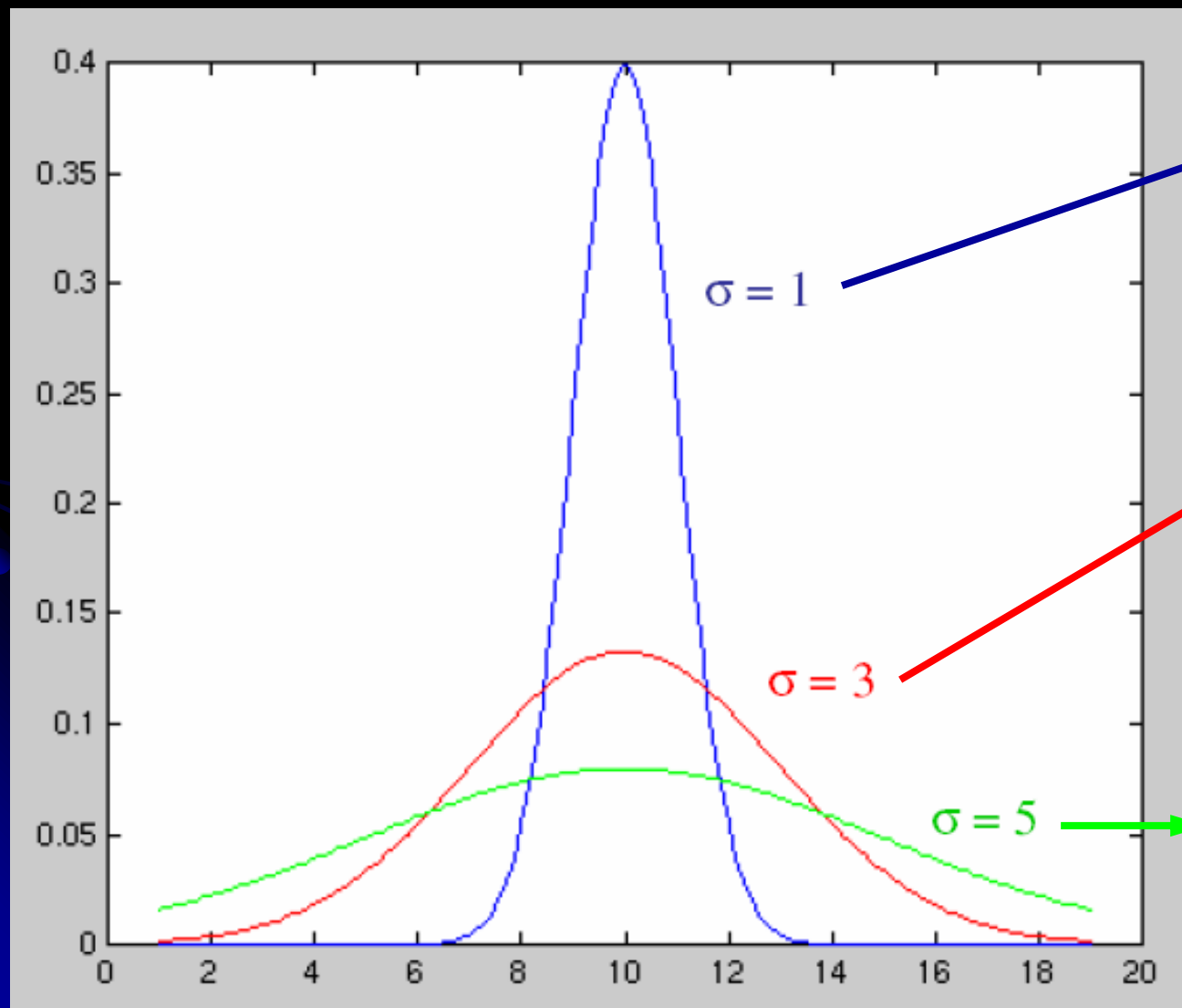


$\sigma = 2$



$\sigma = 4$

Uklanjanje šuma Gauss filterom

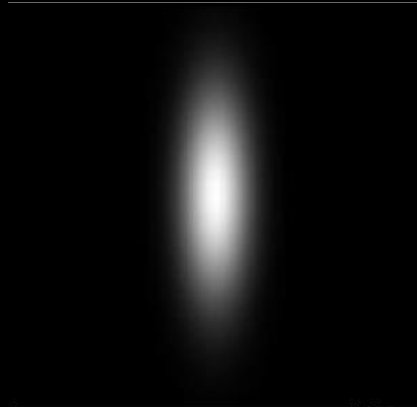


Orientisani Gauss filteri

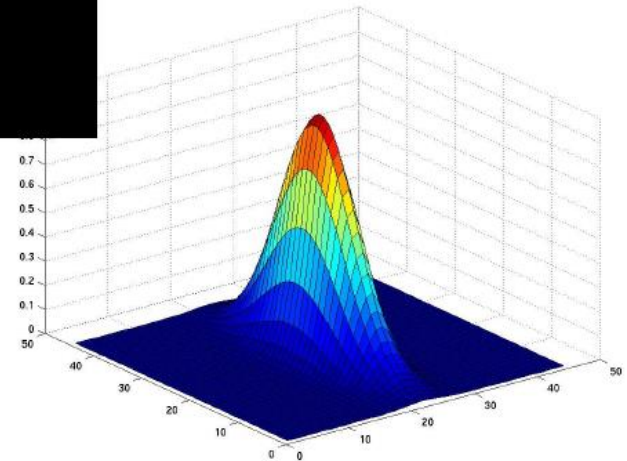
- G_σ omekšava sliku za istu vrijednost u svim smjerovima
- Moguće je omekšati sliku za određenu vrijednost u smjeru definisanim jediničnim vektorom $[a \ b]$ a za drugu vrijednost u smjeru definisanim sa $[c \ d]$

$$G = e^{-\frac{(ax+by)^2}{2\sigma_1^2} - \frac{(cx+dy)^2}{2\sigma_2^2}}$$

Orientisani Gauss filteri

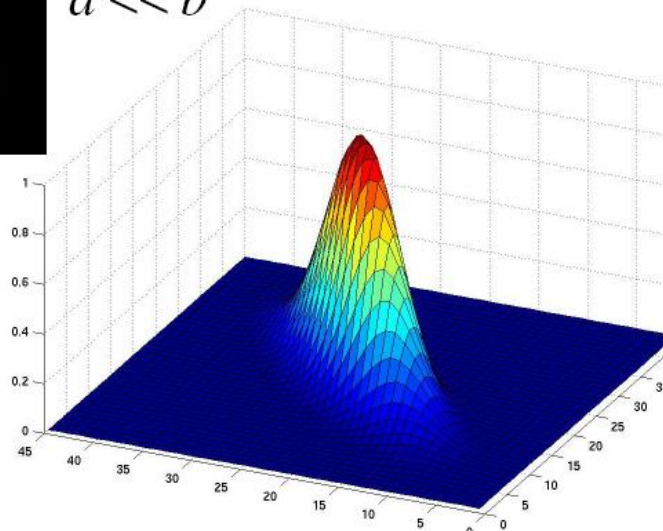


$$\Sigma = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \quad a \ll b$$

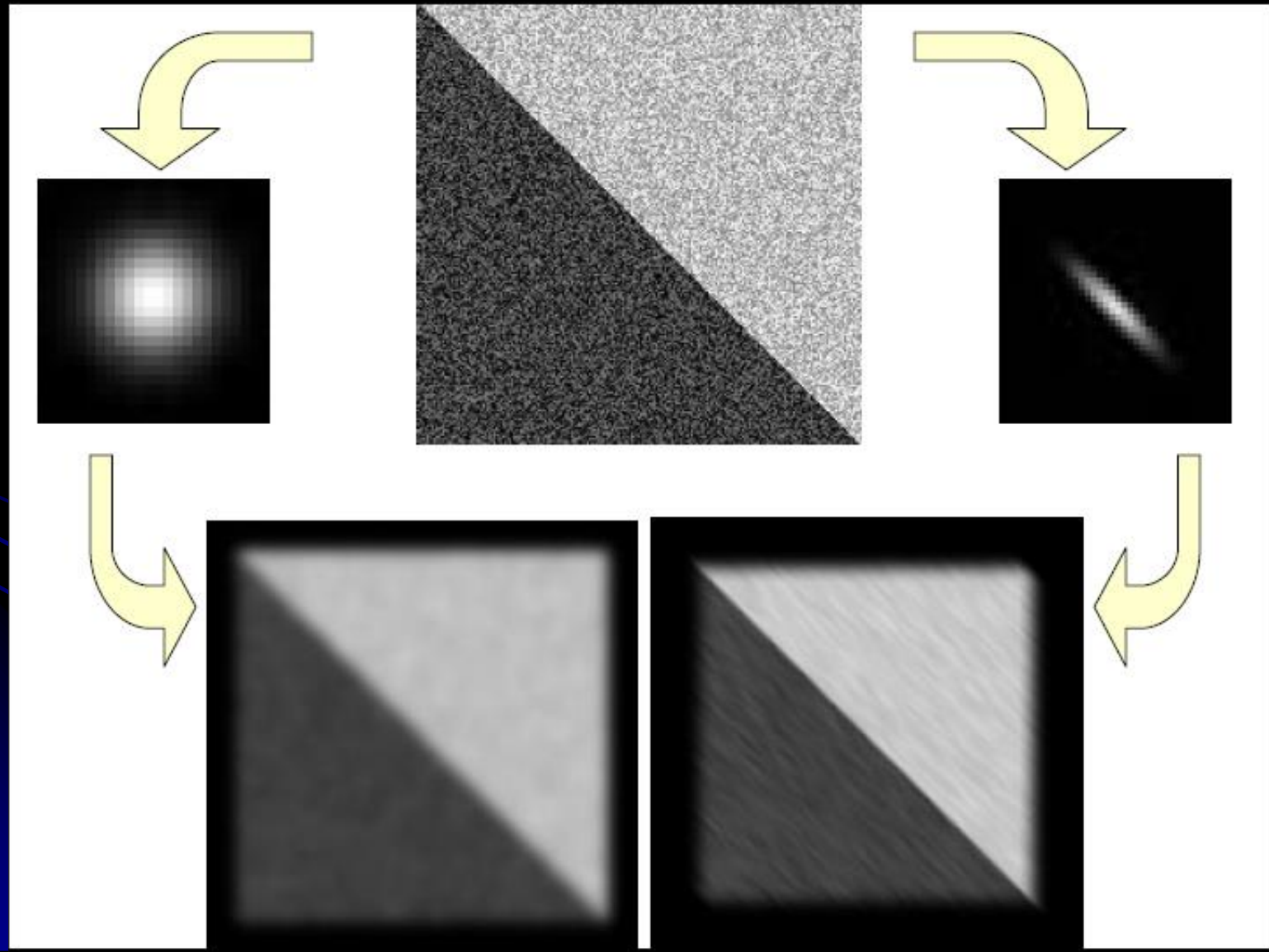


$$\Sigma = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$a \ll b$



Orientisani Gauss filteri



Derivacijski filteri

- Za svaki piksel (x,y) izračunavaju se derivacije (razlika između lijevog i desnog piksela):

$$\frac{\partial I}{\partial x} \approx I(i+1, j) - I(i-1, j)$$

- Konvolucija slike za

$$\partial_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

- Nedostatak: povećava se šum

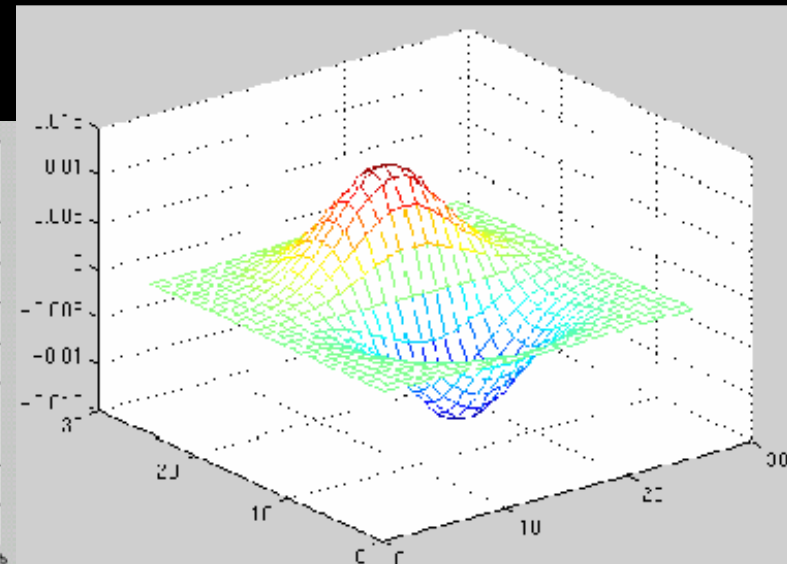
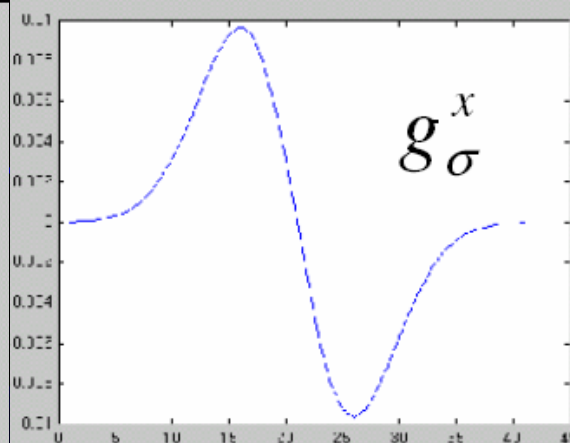
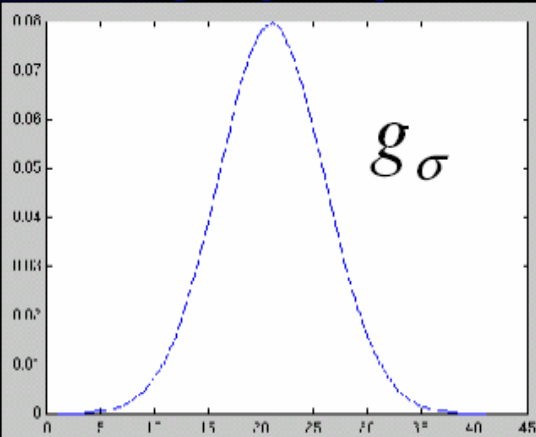
Derivacijski filteri

- Rješenje: prvo omekšati sliku Gauss filterom, a zatim derivirati:
- Koristi se osobina derivacije konvolucije:

$$\frac{\partial f}{\partial x} \approx \frac{\partial (G_\sigma * f)}{\partial x}$$

$$\frac{\partial f}{\partial x} \approx \frac{\partial G_\sigma}{\partial x} * f$$

$$G_\sigma^x = \frac{\partial G_\sigma}{\partial x} = x e^{-\frac{x^2+y^2}{2\sigma^2}}$$



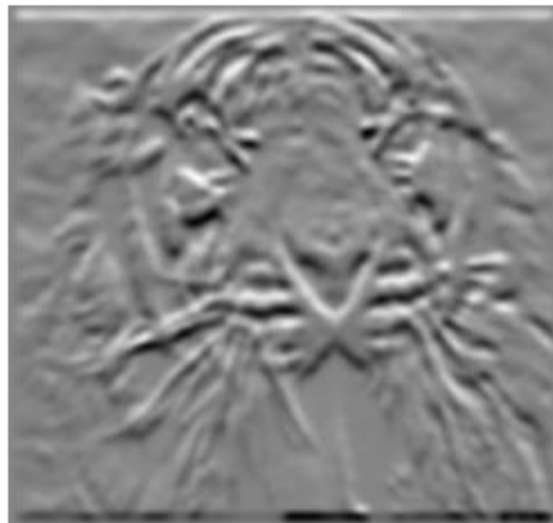
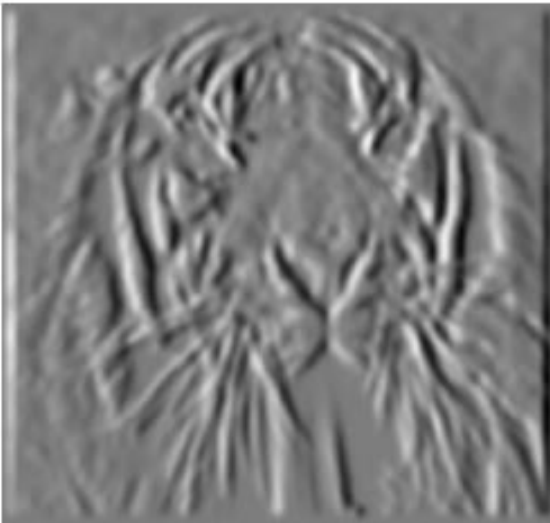
Derivacijski filteri

I



$$|\nabla I| = \sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

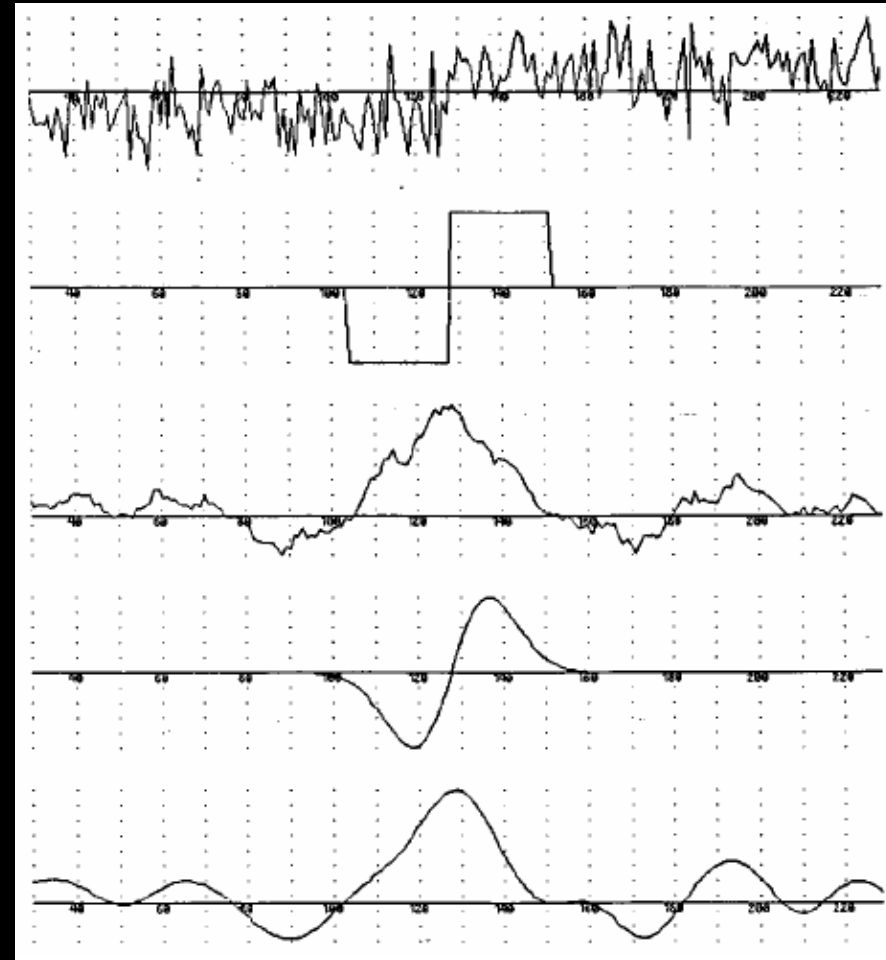
$\frac{\partial I}{\partial x}$



$\frac{\partial I}{\partial y}$

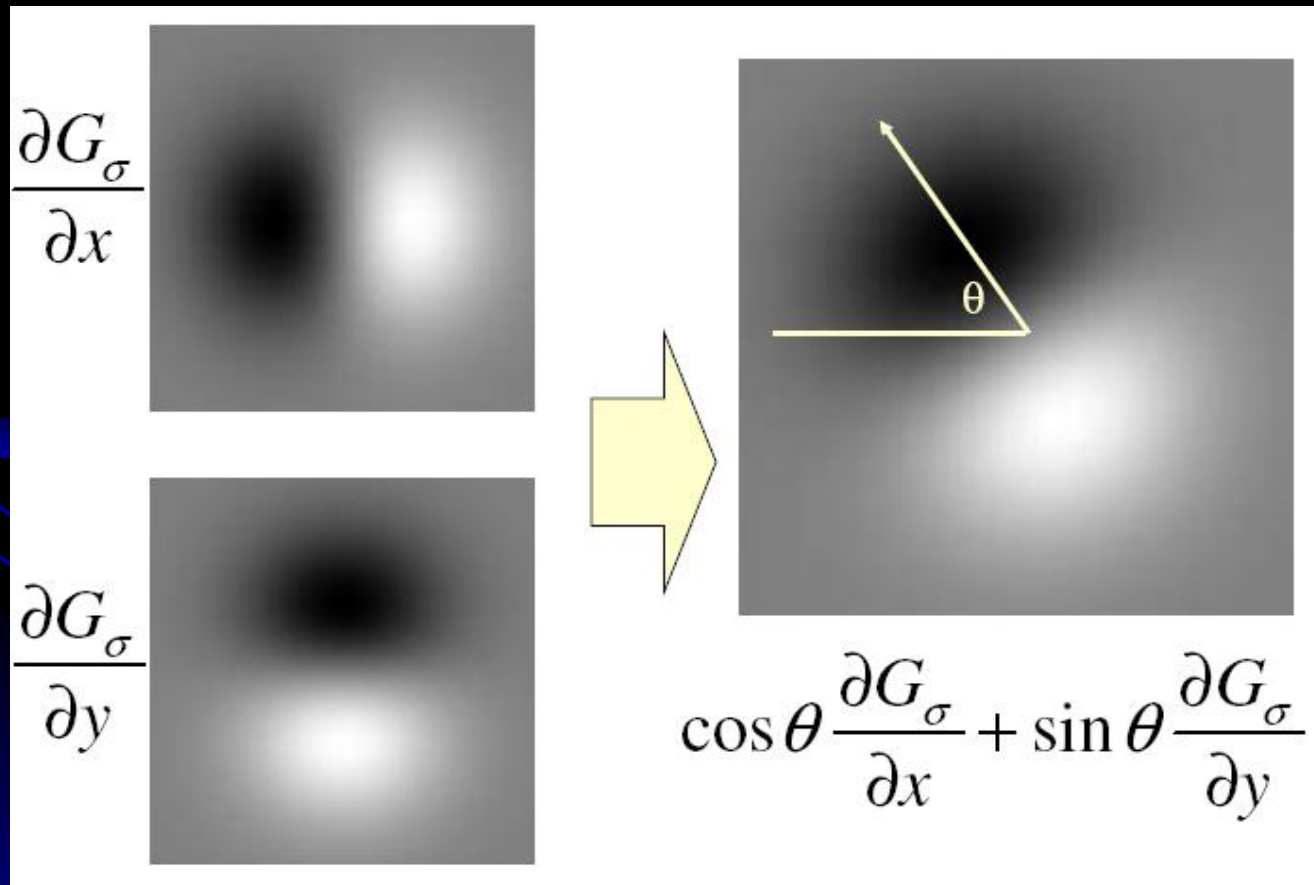
Derivacijski filteri

- Ulaz
- Diferencijalni operator
- Derivacija diferencijalnog operatora
- Derivacija Gauss operatora
- Derivacija derivacije Gauss operatora



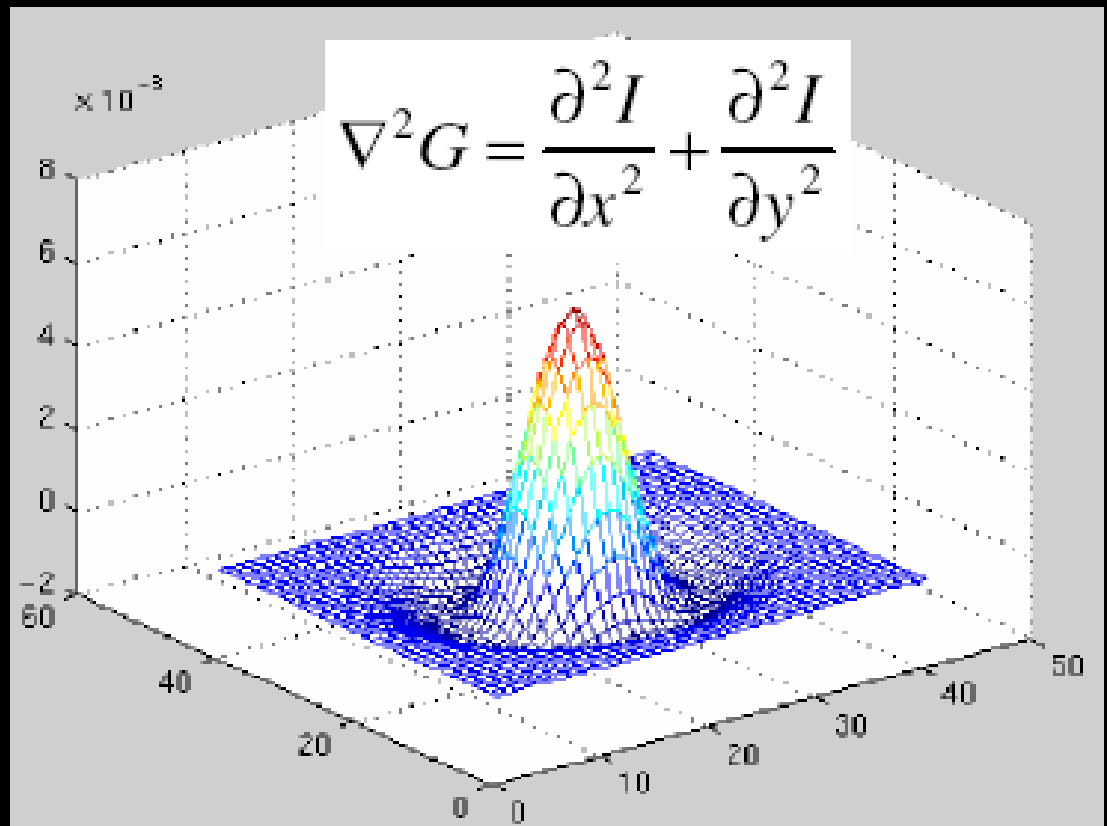
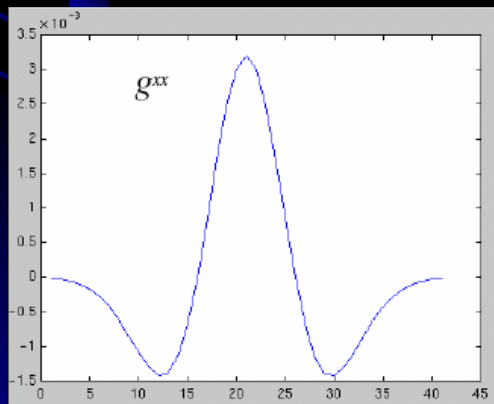
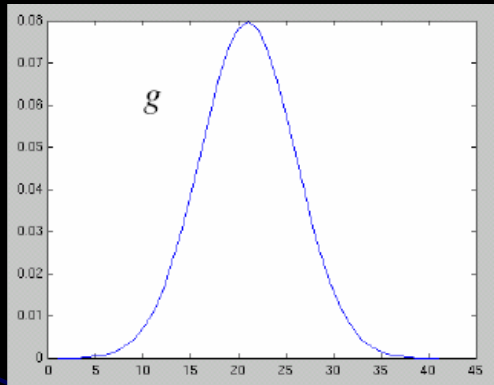
Derivacijski filteri

- I derivacija se može vršiti u smjeru vektora



Derivacijski filteri

- Druga derivacija: Laplacian



Detekcija ivica - vektorizacija



Detekcija ivica

- Ivica = diskontinuitet intenziteta u određenom pravcu.
- Može se otkriti traženjem mjesta gdje su derivacije slike velikih vrijednosti.

